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FORECASTING BITCOIN VOLATILITY USING TWO-COMPONENT CARR MODEL

Abstract. *In this paper, we propose an extension of the range-based CARR model, the two-component CARR (CCARR) model to model and forecast the Bitcoin volatility. The extension inherits the strength of the original range-based CARR model, its capability of exploiting intraday information from the high and low prices to estimate volatility. Moreover, the CCARR model has the capacity to accommodate the long memory volatility. Empirical results show that the CCARR model outperforms the CARR model and the return-based GARCH and two-component GARCH (CGARCH) models in forecasting the Bitcoin volatility. The results highlight the value of using price range and including a second component of the conditional range for forecasting the Bitcoin volatility.*

Keywords*: Bitcoin,Two-component CARR, Price range, Two-component GARCH, Volatility forecasting, Long memory.*

JEL Classification: C5, C22, G17

1. Introduction

Bitcoin was initially introduced in 2008 by Satoshi Nakamoto, which is a digital decentralized cryptocurrency based on the block chain technology (Nakamoto, 2008). Since its creation, Bitcoin has witnessed a rapid development

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and has attracted increasing attention from investors, practitioners and researchers. Over the last few years, the price of Bitcoin has increased tremendously. At the same time, it has experienced extreme volatility. Therefore, it is important to model and forecast the Bitcoin volatility, as it plays a crucial role in investment decision-making and risk management.

Previous studies of the Bitcoin volatility are mainly based on the GARCH-type models. Dyhrberg (2016) employs the GARCH models to study the financial asset capabilities of Bitcoin. Further, Baur et al. (2018) replicate and extend the findings of Dyhrberg (2016). Katsiampa (2017) explores the optimal GARCH model to explain the Bitcoin volatility. Chu et al. (2017) use the GARCH-type models to model seven most popular cryptocurrencies including the Bitcoin. Mensi et al. (2019) use the GARCH models to investigate the impact of dual long memory and structural breaks on the volatility of Bitcoin market. Ardia et al. (2019) use the Markov-switching GARCH models to examine the presence of regime changes in the Bitcoin volatility. Gronwald (2019) employs a GARCH-Jump model to investigate the extreme price movements in the Bitcoin market. Conrad et al. (2018) and Walther et al. (2019) apply the GARCH-MIDAS model to model and forecast the Bitcoin volatility. Köchling et al. (2019) evaluate the volatility forecasting accuracy for Bitcoin using 172 GARCH-type models. Troster et al. (2019) and Trucíos (2019) adopt GARCH models to model and forecast the Bitcoin risk.

The above GARCH approach for studying the volatility dynamics of Bitcoin relies on the daily return data. However, the daily return data are computed from the closing prices, which neglects all intraday price movement. An alternative approach for estimating volatility is to employ the daily price range, which is based on the intraday high and low prices. It is clear that the price range includes more information on intra-period trajectory of the price, whereas the return-based volatility estimator only includes a single measurement of the closing price. Parkinson (1980) and Alizadeh et al. (2002) have shown that the price range is a more efficient volatility estimator than the commonly used return-based one. More recently, Degiannakis and Livada (2013) show that the price range volatility estimator is more accurate than the realized volatility estimator based on five, or less, intraday returns.

Strangely, despite the superiority of the price range, there are very few works using it to study the volatility dynamics of Bitcoin. This paper aims to estimate the Bitcoin volatility using the price range. To capture the dynamics of the price range, Chou (2005) develops the conditional autoregressive range (CARR) model, which has the similar structure with the GARCH model. However, unlike the GARCH model that uses only closing prices data, the CARR model uses intraday data of the high and low prices, which exploits the intraday information to model and estimate volatility. Chou (2005) and Chou and Liu (2010) demonstrate that the CARR model provides more accurate volatility estimates than the standard GARCH model. As a consequence, the CARR model has attracted a great deal of

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attention in the literature (see, e.g., Chen et al., 2008; Chiang and Wang, 2011; Lin et al., 2012; Sin, 2013; Anderson et al., 2015; Auer, 2016; Ng et al., 2017; Xie and Wu, 2017, 2019; Chan et al., 2019; Xie, 2019).

Nevertheless, the standard CARR model may still not be adequate to capture the long memory volatility, a well-known feature of volatility that is important for empirical applications such as volatility forecasting. To address this salient stylized fact of volatility, Engle and Lee (1999) propose a two-component GARCH (CGARCH) model that decomposes volatility into a short-run and a long-run component. Recently, based on the CGARCH model, Katsiampa (2017) shows the importance of including both a short-run and a long-run component of the conditional variance for estimating the volatility of Bitcoin. Motivated by the above interpretation, we extend the standard CARR model to the two-component CARR (CCARR) model to model and forecast the Bitcoin volatility. The CCARR model has the similar structure with the CGARCH model of Engle and Lee (1999), which is able to account for the long-memory volatility and is also simple to implement.

This paper contributes to the literature by modelling and forecasting the Bitcoin volatility using the range-based CARR and CCARR models. The application of return-based GARCH models to model and forecast the Bitcoin volatility has been extensively investigated. However, to the best of our knowledge, there is no previous study applying the CARR model and in particular, the CCARR model to model and forecast the Bitcoin volatility. We examine and compare the out-of-sample forecasting performance of the return-based GARCH and CGARCH models and the range-based CARR and CCARR models. Our results show that the CCARR model outperforms the CARR, GARCH and CGARCH models in forecasting the Bitcoin volatility, highlighting the value of using price range and including a second component of the conditional range for forecasting the Bitcoin volatility.

The rest of the paper is organized as follows. Section 2 describes the volatility models for modelling and forecasting the Bitcoin volatility, namely the return-based GARCH and CGARCH models and the range-based CARR and CCARR models. Section 3 illustrates the forecast evaluation method. Section 4 presents the empirical results, and Section 5 concludes.

2. Volatility Models

In this section, we describe two classes of volatility models, the return-based GARCH and CGARCH models and the range-based CARR and CCARR models, which are used to model and forecast the Bitcoin volatility.

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2.1The GARCH and CGARCH Models

The GARCH model has been widely used in the literature to model and forecast the financial volatility. The standard GARCH(1,1) model proposed by Bollerslev (1986) is given by

$$r_t = \sigma_t z_t, \quad z_t \sim \text{i.i.d. } N(0,1) \tag{1}$$

$$\sigma_t^2 = \omega + \alpha r_{t-1}^2 + \beta \sigma_{t-1}^2 \tag{2}$$

where r_t is the log-return at time t, σ_t^2 is the conditional variance (or squared volatility) of the return r_t based on the information set F_{t-1} up to time $t-1$, or $\sigma_t^2 = \mathrm{Var}[r_t \mid \mathrm{F}_{t-1}]$, z_t is the (standardized) return innovation, which is assumed to follow a standard normal distribution, and ω, α and β are parameters. For the conditional variance σ_t^2 to be positive and stationary, we require that $\omega > 0$, $\alpha, \beta \geq 0$ and $\alpha + \beta < 1$.

It has been well-documented empirically in the finance literature that the financial volatility responds asymmetrically to the positive and negative returns. A popular explanation for this stylized fact is the leverage effect. To capture the leverage effect, alternative asymmetric GARCH models such as the EGARCH and the GJR-GARCH could be employed. However, it has been documented in the literature that the leverage effect is not significant for cryptocurrencies including the Bitcoin (Tiwari et al., 2019). Thus, in the paper we use the standard GARCH model without leverage effect to model and forecast the Bitcoin volatility.

To address the long memory property of volatility, Engle and Lee (1999) extend the standard GARCH model to the CGARCH model, which is given by

$$r_t = \sigma_t z_t, \quad z_t \sim \text{i.i.d. } N(0,1) \tag{3}$$

$$\sigma_t^2 = q_t^2 + \alpha_1 (r_{t-1}^2 - q_{t-1}^2) + \beta_1 (\sigma_{t-1}^2 - q_{t-1}^2) \tag{4}$$

$$q_t^2 = \omega + \alpha_2 (r_{t-1}^2 - \sigma_{t-1}^2) + \beta_2 q_{t-1}^2 \tag{5}$$

where q_t^2 is referred to as the long-run component of the conditional variance and $\sigma_t^2 - q_t^2$ is referred to as the short-run component.

2.2 The CARR and CCARR Models

The GARCH model uses daily return computed from the closing prices to model and estimate volatility. In the paper, we also use the price range which is based on the intraday high and low prices to estimate the Bitcoin volatility. The price range is defined as

$$R_t = \frac{1}{\sqrt{4 \log 2}} (\log H_t - \log L_t) \tag{6}$$

where H_t and L_t denote the high and low prices of Bitcoin at day t, respectively. Parkinson (1980) demonstrates that the price range defined in Eq. (6) is a five times more efficient volatility estimator than the squared return.

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To model the dynamics of the price range, Chou (2005) develops the CARR model, which can be written as

$$R_t = \lambda_t \varepsilon_t, \quad \varepsilon_t \sim \text{i.i.d.}\exp(1) \tag{7}$$

$$\lambda_t = \omega + \alpha R_{t-1} + \beta \lambda_{t-1} \tag{8}$$

where λ_t is the conditional mean of the price range R_t based on the information set F_{t-1} up to time $t-1$, or $\lambda_t = E[R_t \mid \mathrm{F}_{t-1}]$. The disturbance term ε_t is assumed to follow an exponential distribution with unit mean. ω, α and β are parameters. For the conditional range λ_t to be positive and stationary, we require that $\omega > 0$, $\alpha, \beta \geq 0$ and $\alpha + \beta < 1$.

Building upon the CGARCH model from Engle and Lee (1999), we derive the CCARR model for modelling and estimating the Bitcoin volatility as

$$R_t = \lambda_t \varepsilon_t, \quad \varepsilon_t \sim \text{i.i.d.}\exp(1) \tag{9}$$

$$\lambda_t = q_t + \alpha_1 (R_{t-1} - q_{t-1}) + \beta_1 (\lambda_{t-1} - q_{t-1}) \tag{10}$$

$$q_t = \omega + \alpha_2 (R_{t-1} - \lambda_{t-1}) + \beta_2 q_{t-1} \tag{11}$$

where q_t is referred to as the long-run component of the conditional range and $\lambda_t - q_t$ is referred to as the short-run component.

3. Forecasting Evaluation

As the volatility is unobservable, we adopt three measures of the ex-post volatility as proxies of the true volatility: the price range (RNG) defined in Eq. (6), the realized volatility (RV) and the realized range-based volatility (RRV). The RV and RRV are defined as follows:

$$RV_t = \sqrt{\sum_{i=1}^{N} (\log P_{t,i} - \log P_{t,i-1})^2} \tag{12}$$

$$RRV_t = \sqrt{\frac{1}{4\log 2} \sum_{i=1}^{N} (\log H_{t,i} - \log L_{t,i})^2} \tag{13}$$

where $P_{t,i}$, $H_{t,i}$ and $L_{t,i}$ are the last, high and low prices over i th interval on day t, respectively. In the paper, we use 5-min intraday return data to compute the RV and RRV measures.

To compare the forecasting performance of the competing models, we use two robust loss functions of Patton (2011), namely the mean squared error (MSE) and the quasi-likelihood (QLIKE). The MSE and QLIKE are robust to imperfect volatility proxy, which are defined as:

$$\text{MSE}: L_t = \left(MV_t - FV_t\right)^2 \tag{14}$$

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$$\text{QLIKE}: L_t = \frac{MV_t}{FV_t} - \log\frac{MV_t}{FV_t} - 1 \tag{15}$$

where $MV_t = RNG_t$, RV_t or RRV_t is measured volatility, and FV_t is forecasted volatility.

Moreover, we use the Diebold-Mariano (1995) test to determine whether there is significant difference between two competing models in forecasting the Bitcoin volatility. To be specific, we test the superiority of model i over model j using a t-test for the coefficient $\mu_{i,j}$ in

$$\varepsilon_{i,t}^2 - \varepsilon_{j,t}^2 = \mu_{i,j} + \eta_t \tag{16}$$

where $\varepsilon_{i,t}$ and $\varepsilon_{j,t}$ are the forecast errors for models i and j, respectively. It is clear that $\mu_{i,j} > 0$ suggests that the model j dominates the model i and vice versa.

4. Empirical Analysis

4.1 Data

For the empirical analysis, we employ the CCARR model to forecast the Bitcoin volatility. The data consists of daily open, high, low and close prices (in US dollars) for Bitcoin traded on Bitstamp, which are obtained from https://bitcoincharts.com/. The sample spans the period from January 1, 2014 to November 4, 2019. The Bitcoin return is computed as $r_t = \log P_t - \log P_{t-1}$, where P_t is the Bitcoin closing price on day t, and the Bitcoin price range is computed using the Eq. (6). Figure 1 shows the time series plots of the Bitcoin daily (closing) prices, returns and price ranges. It is evident from the figure that the price of Bitcoin increased tremendously with extreme volatility since 2017.

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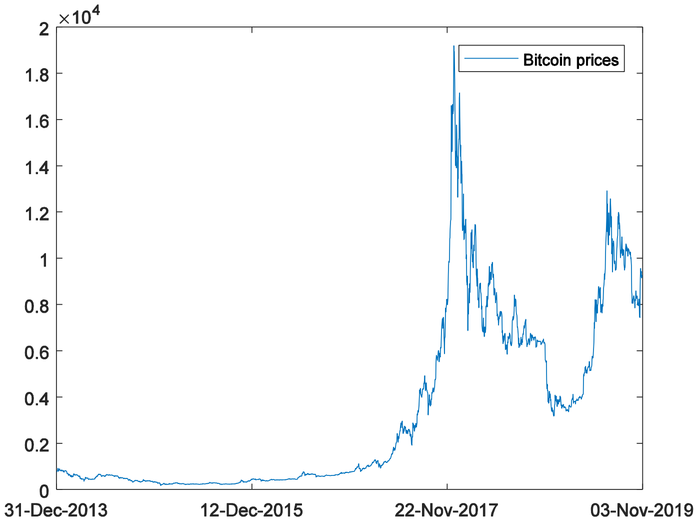

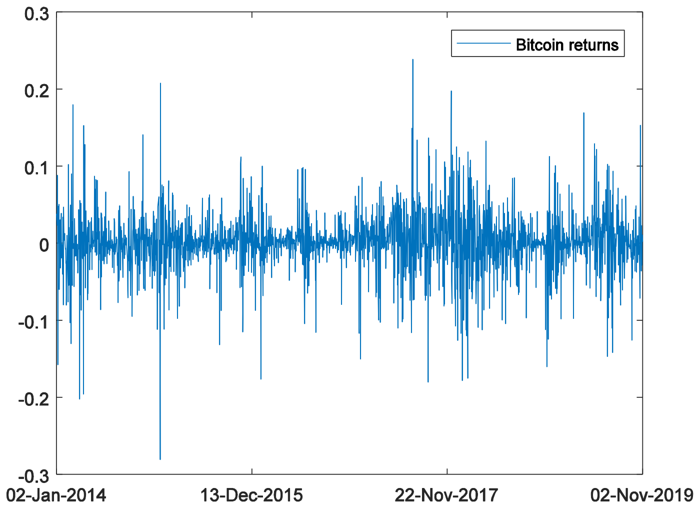

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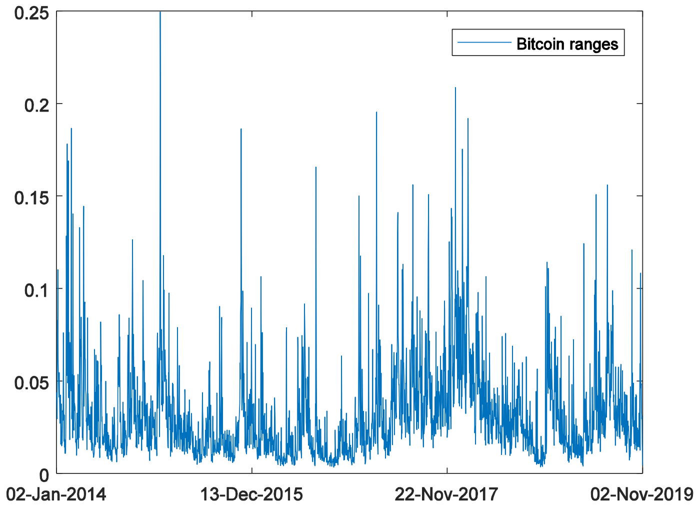

Figure 1. Time series plots of Bitcoin daily prices, returns and price ranges

Table 1 presents descriptive statistics of the Bitcoin daily returns and price ranges. As can be seen from the table, Bitcoin is highly volatile, with an annualized standard deviation of 76.04% ($=0.0398\times\sqrt{365}$) for the returns and an annualized mean of 62.86% ($=0.0329\times\sqrt{365}$) for the price ranges. Also, we find that the Bitcoin returns are negative skewed and leptokurtic, while the Bitcoin ranges are heavily positive skewed and leptokurtic.[1]Moreover, the large Ljung-Box $Q(20)$ statistic for the Bitcoin price ranges shows the Bitcoin volatility exhibits high persistence or long memory behavior.

Table 1.Descriptive statistics of Bitcoin daily returns and price ranges

	Returns	Ranges
Obs.	2127	2127
Mean	0.0012	0.0329
Min.	-0.2809	0.0034
Max.	0.2384	0.2495

[1]In the paper, we use the simple normal distribution in the return-based GARCH and CGARCH models and the exponential distribution in the range-based CARR and CCARR models. Considering the empirical features of the Bitcoin return and range distributions, alternative distribution specifications in the models could be employed. As the focus of the paper is on Bitcoin volatility forecasting rather than distribution choices, we leave it for future research.

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Std. Dev.	0.0398	0.0272
Skewness	-0.2826	2.3193
Kurtosis	8.4388	11.1486
$Q(20)$	37.4641	4234.8936

Note: $Q(20)$ is the Ljung-Box Q statistic for autocorrelation up to lag 20.

4.2 Out-of-Sample Results

In this section, we examine and compare the out-of-sample forecast performance between the GARCH, CGARCH, CARR and CCARR models for forecasting 1-day, 5-days, 10-days, 15-days and 20-days ahead Bitcoin volatility. We perform the forecasts using a rolling window scheme with a fixed window size of 1400 trading days. The rolling window is rolled forward daily. The parameters of the four models are estimated by employing the quasi-maximum likelihood estimation method. The estimation results for the models are not presented here to save spacebut are available upon request.

Tables 2-4 present the out-of-sample forecast results for the four models for the three measured volatilities MV: RNG, RV and RRV. As can be seen from the tables, the range-based (C)CARR model outperforms the return-based (C)GARCH model in most cases for the three measured volatilities and the five forecast horizons in terms of the MSE and QLIKE loss functions, which highlights the value of using price range for modelling and forecasting the Bitcoin volatility. In addition, we observe that the CGARCH/CCARR model outperforms the GARCH/CARR model in most cases. This result demonstrates that including a second volatility component is important for improving the Bitcoin volatility forecasts. It is also worth noting that with the increase of the forecast horizon, the Bitcoin volatility becomes more difficult to forecast for all the four models, which can also be seen clearly from Figure 2. Overall, the CCARR model generally exhibits the lowest MSE and QLIKE losses and appears as the best model in forecasting the Bitcoin volatility.

Table 2.Out-of-sample forecasting results for Bitcoin (MV:RNG)

Horizon	GARCH	CGARCH	CARR	CCARR
MSE				
1	5.3440E-04	5.1814E-04	4.8415E-04	4.7381E-04
5	7.0682E-04	6.9227E-04	6.3162E-04	5.9532E-04
10	7.8296E-04	7.6850E-04	6.8468E-04	6.3429E-04
15	8.1394E-04	7.9551E-04	6.9593E-04	6.5015E-04
20	9.2046E-04	8.8370E-04	7.2960E-04	6.7751E-04
QLIKE				
1	1.6440E-01	1.6119E-01	1.5424E-01	1.5217E-01
5	2.0356E-01	2.0199E-01	1.8557E-01	1.8061E-01

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10	2.2905E-01	2.2965E-01	2.1052E-01	2.0739E-01
15	2.4428E-01	2.4517E-01	2.1745E-01	2.2009E-01
20	2.6269E-01	2.6161E-01	2.2655E-01	2.2829E-01

Table 3.Out-of-sample forecasting results for Bitcoin (MV:RV)

Horizon	GARCH	CGARCH	CARR	CCARR
MSE				
1	4.2051E-04	4.0311E-04	3.7791E-04	3.6090E-04
5	6.1028E-04	5.9783E-04	6.0769E-04	5.5758E-04
10	6.7710E-04	6.6427E-04	6.8147E-04	6.0097E-04
15	6.9180E-04	6.7796E-04	7.0863E-04	6.2376E-04
20	7.9753E-04	7.6621E-04	7.5447E-04	6.5918E-04
QLIKE				
1	9.2612E-02	8.9491E-02	8.8570E-02	8.5380E-02
5	1.3645E-01	1.3423E-01	1.3677E-01	1.2800E-01
10	1.6053E-01	1.6001E-01	1.6339E-01	1.5463E-01
15	1.7113E-01	1.7156E-01	1.7169E-01	1.6709E-01
20	1.8734E-01	1.8660E-01	1.8240E-01	1.7601E-01

Table 4.Out-of-sample forecasting results for Bitcoin (MV:RRV)

Horizon	GARCH	CGARCH	CARR	CCARR
MSE				
1	3.1053E-04	2.9390E-04	2.6445E-04	2.4871E-04
5	4.9328E-04	4.8039E-04	4.6660E-04	4.2227E-04
10	5.6734E-04	5.5274E-04	5.3460E-04	4.6518E-04
15	5.8402E-04	5.6770E-04	5.5531E-04	4.8460E-04
20	6.8742E-04	6.5321E-04	5.9672E-04	5.1711E-04
QLIKE				
1	7.4878E-02	7.1751E-02	6.7023E-02	6.4334E-02
5	1.1865E-01	1.1646E-01	1.1377E-01	1.0594E-01
10	1.4367E-01	1.4297E-01	1.3953E-01	1.3197E-01
15	1.5424E-01	1.5428E-01	1.4642E-01	1.4305E-01
20	1.7079E-01	1.6939E-01	1.5669E-01	1.5179E-01

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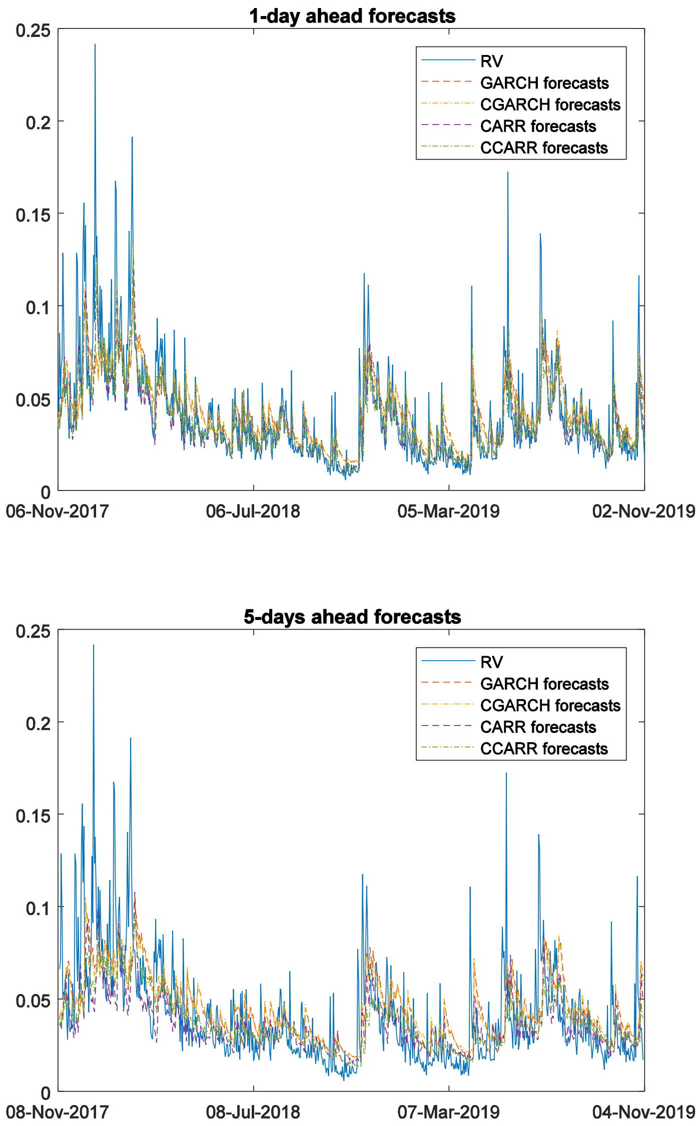

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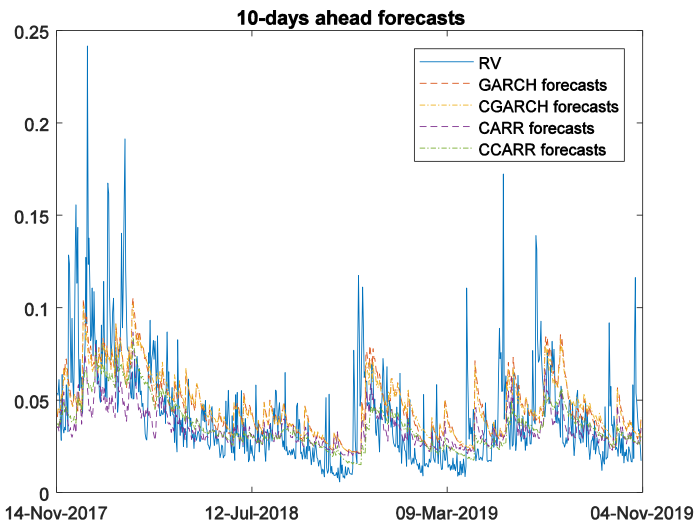

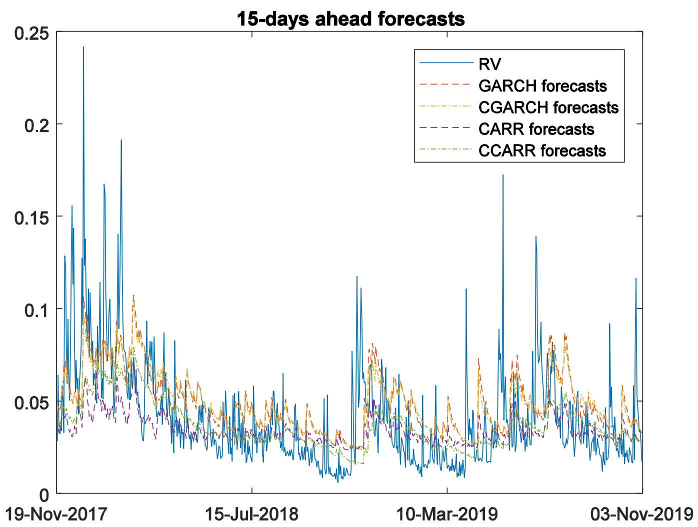

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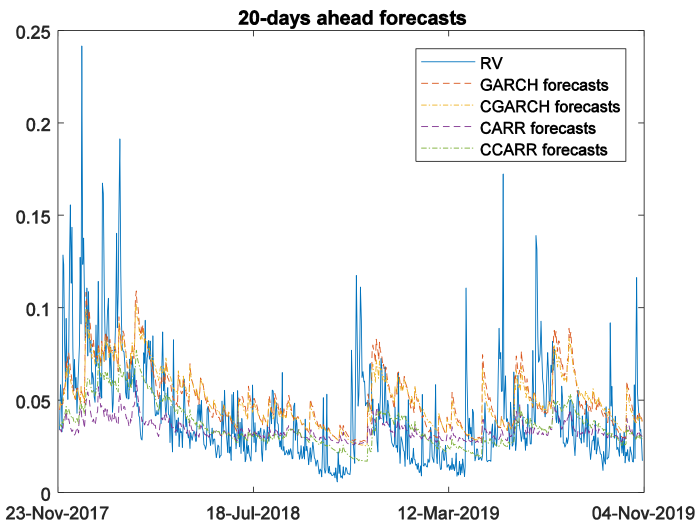

Figure 2.Out-of-sample forecasts of Bitcoin volatility

Further, to see if there is significant difference in volatility forecasting accuracy for Bitcoin between two competing models, we report the Diebold-Mariano test results in Tables5-7. As can be seen from the tables, the Diebold-Mariano statistics for the CGARCH and GARCH models are unanimously reported to be positive and significant, suggesting that the CGARCH model is significantly better than the GARCH model. Most importantly, the Diebold-Mariano statistics for the equality of forecast accuracy of the CCARR forecasts and the others (GARCH, CGARCH and CARR forecasts) are consistently reported to be positive and significant, suggesting that the CCARR model produces significant more accurate out-of-sample forecasts for Bitcoin volatility compared to the other models.

Table 5.Diebold-Mariano test results (MV:RNG)

	CARR	CGARCH	GARCH
Forecast horizon: 1 day			
CCARR	1.7039*	2.8354***	3.6683***
CARR		2.1147**	3.0293***
CGARCH			3.4061***
Forecast horizon: 5 days			
CCARR	2.6963***	4.8644***	5.2377***
CARR		2.6715***	3.1708***
CGARCH			3.0588***
Forecast horizon: 10 days			

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CCARR	3.0873***	5.8963***	5.7500***
CARR		2.7332***	3.0082***
CGARCH			2.2947**
Forecast horizon: 15 days			
CCARR	2.7390***	4.9165***	4.8974***
CARR		2.5529**	2.7983***
CGARCH			2.5394**
Forecast horizon: 20 days			
CCARR	2.8186***	7.2221***	7.2655***
CARR		3.8232***	4.3102***
CGARCH			4.1788***

Note: The Diebold-Mariano test uses a Newey-West type estimator to account for the heteroskedasticity and autocorrelation in the loss differential. A positive statistic suggests that the model in the row dominates the model in the column and vice versa. *, **, and *** stand for statistical significance at the 10%, 5%, and 1% levels, respectively.

Table 6.Diebold-Mariano test results (MV:RV)

	CARR	CGARCH	GARCH
Forecast horizon: 1 day			
CCARR	3.0603***	2.7936***	3.9466***
CARR		1.6387	2.8336***
CGARCH			3.8684***
Forecast horizon: 5 days			
CCARR	3.5636***	2.2521**	2.7227***
CARR		-0.4294	0.1070
CGARCH			2.7498***
Forecast horizon: 10 days			
CCARR	4.6983***	2.9684***	3.0444***
CARR		-0.5517	-0.1294
CGARCH			2.0187**
Forecast horizon: 15 days			
CCARR	4.9785***	1.7347*	1.9160*
CARR		-0.7537	-0.3795
CGARCH			2.0441**
Forecast horizon: 20 days			
CCARR	4.9893***	3.9710***	4.4364***
CARR		0.2956	0.9986
CGARCH			4.1649***

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Note: The Diebold-Mariano test uses a Newey-West type estimator to account for the heteroskedasticity and autocorrelation in the loss differential. A positive statistic suggests that the model in the row dominates the model in the column and vice versa. *, **, and *** stand for statistical significance at the 10%, 5%, and 1% levels, respectively.

Table 7.Diebold-Mariano test results (MV:RRV)

	CARR	CGARCH	GARCH
Forecast horizon: 1 day			
CCARR	3.3844***	3.6531***	4.9522***
CARR		2.3484**	3.7237***
CGARCH			4.4179***
Forecast horizon: 5 days			
CCARR	3.6141***	3.7304***	4.1984***
CARR		0.6909	1.2697
CGARCH			3.1855***
Forecast horizon: 10 days			
CCARR	4.6100***	4.5941***	4.5860***
CARR		0.6593	1.0991
CGARCH			2.5900***
Forecast horizon: 15 days			
CCARR	4.6671***	3.0810***	3.2382***
CARR		0.3495	0.7423
CGARCH			2.6263***
Forecast horizon: 20 days			
CCARR	4.6963***	5.5936***	5.9774***
CARR		1.5967	2.3393**
CGARCH			4.6735***

Note: The Diebold-Mariano test uses a Newey-West type estimator to account for the heteroskedasticity and autocorrelation in the loss differential. A positive statistic suggests that the model in the row dominates the model in the column and vice versa. *, **, and *** stand for statistical significance at the 10%, 5%, and 1% levels, respectively.

5. Conclusions

Bitcoin has attracted increasing attention in the recent years, and the price of Bitcoin has increased tremendously with extreme volatility. Therefore, it is of great importance to model and forecast the Bitcoin volatility, as it is crucial for investor's decision making and risk management. However, most of the previous

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studies focus on GARCH-type models, which are return-based models that use only information from the closing prices to model and estimate the Bitcoin volatility. An alternative approach for modelling and forecasting the Bitcoin volatility is to employ the range-based volatility models that exploit the intraday information from the high and low prices.

In this paper, the range-based CARR and CCARR models are used to model and forecast the Bitcoin volatility. To the best of our knowledge, this is the first study to use the CARR and CCARR models to model and forecast the Bitcoin volatility. We investigate and compare the out-of-sample performance between the return-based GARCH and CGARCH and range-based CARR and CCARR models for forecasting 1-day, 5-days, 10-days, 15-days and 20-days ahead Bitcoin volatility. We adopt two robust loss functions, namely the mean squared error (MSE) and the quasi-likelihood (QLIKE), as well as the Diebold-Mariano test to evaluate the out-of-sample performance of the competing models. The out-of-sample results show that the CCARR model generates more accurate out-of-sample forecasts of Bitcoin volatility compared to the GARCH, CGARCH and CARR models under various volatility proxies, which highlights the value of using price range and including a second component of the conditional range for forecasting the Bitcoin volatility. Our findings have important implications for Bitcoin allocations and risk management, and can help investors make more reasonable decisions.

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